# Semantic Theory Lecture 14: Discourse Semantics II 

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## Discourse Representation Theory



## An Example

- A farmer owns a donkey. He beats it.

$$
\begin{aligned}
& x y \operatorname{z~u} \\
& \hline \text { farmer(x) } \\
& \operatorname{donkey}(y) \\
& \text { owns }(x, y) \\
& z=x \\
& u=y \\
& \operatorname{beat}(z, u)
\end{aligned}
$$

## Denotational Interpretation

- Let
- $K=\left\langle U_{k}, C_{k}\right\rangle$ a DRS
- $M=\left\langle U_{M}, V_{M}\right\rangle$ a FOL model structure appropriate for $K$ (i.e., M provides interpretations for all relation symbols occurring in K).
- An embedding of $K$ into $M$ is a function $f$ from $U_{K}$ to $U_{M}$.


## Verifying embedding

- An embedding $f$ of $K$ in $M$ verifies $K$ in $M$ iff $f$ verifies every condition $\alpha \in C_{k}$
- Notation: $\mathrm{f} \mathrm{F}_{\mathrm{M}} \mathrm{K}$
- f verifies condition $\alpha$ in $M\left(f \models_{M} \alpha\right)$ :
- $f \vDash_{M} R\left(x_{1}, \ldots, x_{n}\right)$ iff $\left\langle f\left(x_{1}\right), \ldots, f\left(x_{n}\right)\right\rangle \in V_{M}(R)$
- $f \vDash_{\mathrm{M}} \mathrm{X}=\mathrm{a}$ iff $\mathrm{f}(\mathrm{x})=\mathrm{V}_{\mathrm{M}}(\mathrm{a})$
- $f \vDash_{\mathrm{M}} \mathrm{x}=\mathrm{y} \quad$ iff $\mathrm{f}(\mathrm{x})=\mathrm{f}(\mathrm{y})$


## Truth

- Let K be a closed DRS and M be an appropriate model structure for $K$.
- $K$ is true in $M$ iff there is a verifying embedding $f$ of $K$ in $M$.


## Verifying Embedding: Example

Let K be the example DRS from above:

■ $K=<\{x, y, z, u\}$,
$\{\operatorname{professor}(x), \operatorname{book}(y), \operatorname{own}(x, y), \operatorname{read}(z, u), z=x, u=y\}>$

■ $f \mid={ }_{M} K$ iff f verifies every condition $\alpha \in C_{K}$, i.e.:

$$
\begin{aligned}
& \left.f\left|=_{M} \operatorname{professor}(x), f\right|_{M} \operatorname{book}(y) f\right|_{M}, \operatorname{own}(x, y), \\
& f\left|\left.\right|_{M} \operatorname{read}(z, u), f\right|=_{M} z=x, \text { and }\left.f\right|_{M} u=y
\end{aligned}
$$

- This holds iff:
$\mathrm{f}(\mathrm{x}) \in \mathrm{V}_{\mathrm{M}}$ (professor), $\mathrm{f}(\mathrm{y}) \in \mathrm{V}_{\mathrm{M}}($ book $),\langle\mathrm{f}(\mathrm{x}), \mathrm{f}(\mathrm{y})\rangle \in \mathrm{V}_{\mathrm{M}}($ own $)$, $\langle f(z), f(u)\rangle \in V_{M}($ read $), f(z)=f(x)$, and $f(u)=f(y)$


## Simplification

- $f(x) \in V_{M}($ professor $) \wedge f(y) \in V_{M}($ book $) \wedge\langle f(x), f(y)\rangle \in V_{M}(o w n) \wedge$ $\langle f(z), f(u)\rangle \in V_{M}(r e a d) \wedge f(z)=f(x) \wedge f(u)=f(y)$
iff
- $f(x) \in V_{M}($ professor $) \wedge f(y) \in V_{M}($ book $) \wedge\langle f(x), f(y)\rangle \in V_{M}($ own $) \wedge$ $\langle f(x), f(u)\rangle \in V_{M}($ read $) \wedge f(u)=f(y)$
- $f(x) \in V_{M}($ professor $) \wedge f(y) \in V_{M}($ book $) \wedge\langle f(x), f(y)\rangle \in V_{M}($ own $) \wedge$ $\langle f(x), f(u)\rangle \in V_{M}(r e a d) \wedge f(u)=f(y)$
iff
- $f(x) \in V_{M}($ professor $) \wedge f(y) \in V_{M}($ book $) \wedge\langle f(x), f(y)\rangle \in V_{M}(o w n) \wedge$ $\langle f(x), f(y)\rangle \in V_{M}($ read $)$


## Truth: Example

- $K=<x, y, z, u\}$, $\{\operatorname{professor}(x), \operatorname{book}(y), \operatorname{own}(x, y), \operatorname{read}(z, u), z=x, u=y\}>$
- Embedding f verifies K in $\mathrm{M}: \mathrm{f} \mid{ }_{=\mathrm{M}} \mathrm{K}$ iff f verifies every condition $\alpha \in C_{K}$ iff $f(x) \in V_{M}($ professor $) \wedge f(y) \in V_{M}($ book $) \wedge\langle f(x), f(y)\rangle \in V_{M}(o w n) \wedge$ $\langle f(x), f(y)\rangle \in V_{M}($ read $)$

■ $K$ is true in $M$ iff there is an $f$ such that $\left.f\right|_{=_{M}} K$

## The Basic Contribution of DRT

- DRT provides an integrated model of global anaphoric relations (through DRS construction) and truth-conditional semantics (through model embedding).
- In particular, DRT models the ambivalent status of indefinite NPs: Indefinite noun phrases introduce new reference objects into context, and at the same time express existential quantification.


## Translation of DRSes to FOL

| $x_{1} \ldots x_{n}$ |
| :--- |
|  |
| $c_{1} \ldots c_{n}$ |

- $\operatorname{DRS} K=\left\langle\left\{x_{1}, \ldots, x_{n}\right\},\left\{c_{1}, \ldots, c_{k}\right\}\right\rangle$
is truth-conditionally equivalent to the FOL formula:

$$
\exists x_{1} \ldots \exists x_{n}\left[c_{1} \wedge \ldots \wedge c_{k}\right]
$$

## Indefinite NPs and Conditionals

(1) If a student works, the professor is happy.
(2) $\exists x[$ student $(x) \wedge$ work(x)] $\rightarrow$ happy(the-professor)
(3) $\forall x[s t u d e n t(x) ~ \wedge ~ w o r k(x) ~ \rightarrow ~ h a p p y(t h e-p r o f e s s o r)] ~$

- Formulas (2) and (3) are logically equivalent:
- $\exists x A \rightarrow B \Leftrightarrow \forall x[A \rightarrow B]$ (provided that variable $x$ does not occur free in $B$ )


## Statives and Non-Statives: Linguistic Evidence

- If a student works, she will be successful.
(1) $\exists x[$ student $(x) \wedge$ work $(x)] \rightarrow$ successful $(x)$
(2) $\exists x[$ student $(x) \wedge$ work $(x) \rightarrow$ successful $(x)]$
(3) $\forall x[$ student $(x) \wedge$ work $(x) \rightarrow$ successful $(x)]$
- Formula (1) is not closed (x occurs free)
- Formula (2) has wrong truth conditions (much too weak)
- Formula (3) is correct, but how can it be derived compositionally?


## Indefinite NPs in Hypothetical Text

- A car is parked in front of Bill's garage. Bill needs to get to the office quickly. He doesn't know who owns the car. He calls the police, and the car is towed away.
- Suppose a car is parked in front of Bill's garage. Bill needs to get to the office quickly. He doesn't know who owns the car. Then he will call the police, and the car will be towed away.
- Let $a$ and $b$ be two positive integers. Let $b$ further be even. Then the product of $a$ and $b$ is even too.


## DRS for Conditionals

- If a professor owns a book, he reads it.



## DRS (1 ${ }^{\text {st }}$ Extension)

- A discourse representation structure (DRS) K is a pair〈 $U_{\kappa}, C_{k}$, where
- $U_{k}$ is a set of discourse referents
- $C_{K}$ is a set of conditions
- (Irreducible) conditions:
- $R\left(u_{1}, \ldots, u_{n}\right) \quad R n$-place relation, $u_{i} \in U_{k}$
- $u=v \quad u, v \in U_{k}$
- $u=a \quad u \in U_{k}, a$ is a proper name
- $K_{1} \Rightarrow K_{2} \quad K_{1}$ and $K_{2}$ DRSs
- Reducible conditions: as before


## Construction Rule for Conditionals

- Triggering configuration:
- $\alpha$ is a reducible condition in DRS K of the form
[s if [s $\beta$ ] (then) [s $\gamma$ ]]
- Action:
- Remove $\alpha$ from $C_{k}$.
- Add $K_{1} \Rightarrow K_{2}$ to $C_{K}$, where
- $K_{1}=\langle\varnothing,\{\beta\}\rangle$
- $K_{2}=\langle\varnothing,\{\gamma\}\rangle$
- Remark: $\mathrm{K}_{1} \Rightarrow \mathrm{~K}_{2}$ is called a duplex condition; $\mathrm{K}_{1}$ the "antecedent DRS" and $K_{2}$ the "consequent DRS."


## An Example

If a professor owns a book, he reads it.


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## An Example

If a professor owns a book, he reads it.


## Embedding: Basic Version

- Let $K=\left\langle U_{K}, C_{K}\right\rangle$ a $D R S, M=\left\langle U_{M}, V_{M}\right\rangle$ an FOL model structure appropriate for $K$. An embedding of $K$ into $M$ is a function from $U_{K}$ to $U_{M}$.
- An embedding $f$ of $K$ into $M$ verifies $K$ in $M$ : $f \models_{m} K$
iff f verifies every condition $\alpha \in \mathrm{C}_{\mathrm{k}}$.
- f verifies condition $\alpha$ in $\mathrm{M}\left(\mathrm{f} \mid{ }_{=_{M}} \alpha\right)$ :
(i) $\mathrm{f} \mid={ }_{\mathrm{M}} \mathrm{R}\left(\mathrm{x}_{1}, \ldots, \mathrm{x}_{\mathrm{n}}\right)$
iff
$\left\langle f\left(x_{1}\right), \ldots, f\left(x_{n}\right)\right\rangle \in V_{M}(R)$
(ii) $f \mid={ }_{M} x=a$
iff $\quad f(x)=V_{M}(a)$
(iii) $f \mid{ }_{m} x=y$
iff $\quad f(x)=f(y)$


## Verifying Embedding for Duplex Conditions (Preliminary)

- f п $_{\text {м }} K_{1} \Rightarrow K_{2}$
iff for all $g$ 〇ukı $f$ such that $g \models_{M} K_{1}$, also $g \models_{M} K_{2}$
- We write $g$ 〇uffor " $g \supseteq f$ and $\operatorname{Dom}(g)=\operatorname{Dom}(f) \cup U "$


## This seems to work ...

If a professor owns a book, he reads it.


## ... but in the general case, it doensn't

- Mary knows a professor. If he owns a book, he gives it to a student.

| su |  |
| :---: | :---: |
| s = Mary professor(u) | know(s, u) |
| $x \mathrm{y}$ | z v w |
| $x=u$ <br> book (y) <br> owns (x, y) | $\Rightarrow \left\lvert\, \begin{aligned} & \text { gives(z,v,w) } \\ & z=x \\ & v=y \\ & \text { student(w) } \end{aligned}\right.$ |

# Verifying embedding for Duplex Conditions 

■ f м $_{\text {м }} \mathbf{K 1} \Rightarrow \mathbf{K 2}$ iff
for all $g \supseteq_{U_{K 1}} f$ such that $g \models_{M} K_{1}$, there is a $\mathrm{h} \supseteq_{U_{K} 2} \mathrm{~g}$ such that $\mathrm{h} \models_{\mathrm{M}} \mathrm{K}_{2}$

## Embedding (Revised)

- Let $U_{D}$ a set of discourse referents,
$\mathrm{K}=\left\langle\mathrm{U}_{\mathrm{K}}, \mathrm{C}_{\mathrm{K}}\right\rangle$ a DRS with $\mathrm{U}_{\mathrm{K}} \subseteq \mathrm{U}_{\mathrm{D}}$,
$\mathrm{M}=\left\langle\mathrm{U}_{\mathrm{M}}, \mathrm{V}_{\mathrm{M}}\right\rangle$ a FOL model structure appropriate for K .
- An embedding of $K$ into $M$ is a (partial) function from $U_{D}$ to $\mathrm{U}_{\mathrm{M}}$ such that $\mathrm{U}_{\mathrm{K}} \subseteq \operatorname{Dom}(\mathrm{f})$.


## Verifying Embedding: $1^{\text {st }}$ Extension

- An embedding $f$ of $K$ into $M$ verifies $K$ in $M: f \models_{M} K$
iff f verifies every condition $\alpha \in \mathrm{C}_{\mathrm{k}}$.
- f verifies condition $\alpha$ in $M\left(f \mid={ }_{m} \alpha\right)$ :
(i) $f \mid={ }_{M} R\left(x_{1}, \ldots, x_{n}\right) \quad$ iff $\quad\left\langle f\left(x_{1}\right), \ldots, f\left(x_{n}\right)\right\rangle \in V_{M}(R)$
(ii) $f \mid={ }_{M} x=a \quad$ iff $\quad f(x)=V_{M}(a)$
(iii) $f \mid={ }_{M} x=y \quad$ iff $f(x)=f(y)$
(iv) $f \vDash_{M} K_{1} \Rightarrow K_{2} \quad$ iff $\quad$ for all $g \supseteq_{U_{1}} f$ such that $g \models_{M} K_{1}$, there is a $\mathrm{h} \supseteq_{U_{K 2}} \mathrm{~g}$ such that $\mathrm{h} \models_{\mathrm{M}} \mathrm{K}_{2}$


## Definition of Truth, Revised

- Let K be a closed DRS and M be an appropriate model structure for $K$.
- $K$ is true in $M$ iff there is a verifying embedding $f$ of $K$ in $M$ such that $\operatorname{Dom}(f)=\mathbf{U}_{\mathrm{K}}$.


## Construction Rule for Universal NPs

- Triggering configuration:
- $\alpha$ is a reducible condition in DRS K; $\alpha$ contains a subtree [s [np $\beta$ ] [vp $\gamma$ ]] or [vp [v $\gamma$ ] [np $\beta$ ]]
- $\beta=$ every $\delta$
- Action:
- Remove $\alpha$ from CK.
- Add $\mathrm{K}_{1} \Rightarrow \mathrm{~K}_{2}$ to $\mathrm{C}_{\mathrm{K}}$, where
- $K_{1}=\langle\{x\},\{\delta(x)\}\rangle$ and
- $\mathrm{K}_{2}=\left\langle\varnothing,\left\{\alpha^{\prime}\right\}\right\rangle$
- obtain $\alpha^{\prime}$ from $\alpha$ by replacing $\beta$ by $x$


## Construction Rule for Negation

- Triggering configuration:
- $\alpha$ is a reducible condition in DRS K of the form
[s $\beta$ [vp doesn't [vp $ү$ ]]]
- Action:
- Remove $\alpha$ from $C_{k}$
- Add $\neg K_{1}$ to $C_{K}$, where $K_{1}=\langle\varnothing$, $\{$ [s $\beta$ [vp $\gamma$ ]] $\}\rangle$


## Negation: Example

- A professor doesn't own a book.



## Negation: Example

- A professor doesn't own a book.



## Negation: Example

- A professor doesn't own a book.



## Negation: Example

- A professor doesn't own a book.



## Negation: Example, $2^{\text {nd }}$ Reading

- A professor doesn't own a book.



## Negation: Example, $2^{\text {nd }}$ Reading

- A professor doesn't own a book.



## Negation: Example, $2^{\text {nd }}$ Reading

- A professor doesn't own a book.



## Construction rule for clausal disjunction

- Triggering configuration:
- $\alpha$ is a reducible condition in DRS $K$ of the form
[s [s $\beta$ ] or [s $\gamma$ ]]
- Action:
- Remove $\alpha$ from $C_{k}$
- Add $K_{1}$ v $K_{2}$ to $C_{k}$, where
- $K_{1}=\langle\varnothing,\{\beta\}\rangle$ and
- $K_{2}=\langle\varnothing,\{\gamma\}\rangle$


## An Example

■ A student reads a book, or a professor reads a paper.


## DRS: $2^{\text {nd }}$ Extension

- A discourse representation structure (DRS) K is a pair〈 $U_{k}, C_{k}$, where
- $U_{k}$ is a set of discourse referents
- $C_{K}$ is a set of conditions
- (Irreducible) conditions:
- $R\left(u_{1}, \ldots, u_{n}\right) \quad R n$-place relation, $u_{i} \in U_{k}$
- $u=v \quad u, v \in U_{k}$
- $u=a \quad u \in U_{k}, a$ is a proper name
- $K_{1} \Rightarrow K_{2}$
$\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ DRSs
- $K_{1} \vee K_{2}$
$\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ DRSs
- $\neg \mathrm{K}_{1}$
$\mathrm{K}_{1}$ DRS


## Verifying Embedding: $2^{\text {nd }}$ Extension

- An embedding $f$ of $K$ into $M$ verifies $K$ in $M: f \models_{M} K$
iff f verifies every condition $\alpha \in \mathrm{C}_{\mathrm{k}}$.
- $f$ verifies condition $\alpha$ in $M\left(f \mid={ }_{M} \alpha\right)$ :
(i) $f \mid={ }_{M} R\left(x_{1}, \ldots, x_{n}\right)$ iff $\left\langle f\left(x_{1}\right), \ldots, f\left(x_{n}\right)\right\rangle \in V_{M}(R)$
(ii) $f \mid={ }_{M} x=a \quad$ iff $\quad f(x)=V_{M}(a)$
(iii) $f \mid=_{M} x=y \quad$ iff $\quad f(x)=f(y)$
(iv) $f \vDash M K 1 \Rightarrow K 2 \quad$ iff $\quad$ for all $g \supseteq_{U_{K 1}} f$ such that $g \models_{M} K_{1}$, there is a $\mathrm{h} \supseteq_{U_{K 2}} \mathrm{~g}$ such that $\mathrm{h} \models_{\mathrm{M}} \mathrm{K}_{2}$
(v) $f \vDash_{\text {м }} K_{1} v K_{2}$ iff there is a $g_{1} \supseteq$ ик $f$ such that $g_{1} \models_{\text {м }} K_{1}$ or there is a $g_{2} \supseteq_{\text {UK2 }} f$ such that $g_{2} \models_{\mathrm{M}} \mathrm{K}_{2}$
(vi) $f \vDash_{M} \neg K_{1} \quad$ iff $\quad$ there is no $g \supseteq_{\text {UK1 }} f$ such that $g \vDash_{M} K_{1}$

